Calculus  
Calculus is the mathematical study of continuous change, in the same way that geometry is the  
study of shape, and algebra is the study of generalizations of arithmetic operations.  
Originally called infinitesimal calculus or "the calculus of infinitesimals", it has two major  
branches, differential calculus and integral calculus. The former concerns instantaneous rates of  
change, and the slopes of curves, while the latter concerns accumulation of quantities, and areas  
under or between curves. These two branches are related to each other by the fundamental  
theorem of calculus. They make use of the fundamental notions of convergence of infinite  
sequences and infinite series to a well-defined limit.[1]  
Infinitesimal calculus was developed independently in the late 17th century by Isaac Newton and  
Gottfried Wilhelm Leibniz.[2][3] Later work, including codifying the idea of limits, put these  
developments on a more solid conceptual footing. Today, calculus has widespread uses in science,  
engineering, and social science.[4]  
In mathematics education, calculus is an abbreviation of both infinitesimal calculus and integral  
calculus, which denotes courses of elementary mathematical analysis.  
In Latin, the word calculus means “small pebble”, (the diminutive of calx, meaning "stone"), a  
meaning which still persists in medicine. Because such pebbles were used for counting out  
distances,[5] tallying votes, and doing abacus arithmetic, the word came to be the Latin word for  
calculation. In this sense, it was used in English at least as early as 1672, several years before the  
publications of Leibniz and Newton, who wrote their mathematical texts in Latin.[6]  
In addition to differential calculus and integral calculus, the term is also used for naming specific  
methods of computation or theories that imply some sort of computation. Examples of this usage  
include propositional calculus, Ricci calculus, calculus of variations, lambda calculus, sequent  
calculus, and process calculus. Furthermore, the term "calculus" has variously been applied in  
ethics and philosophy, for such systems as Bentham's felicific calculus, and the ethical calculus.  
Etymology  
History

Archimedes used the  
method of exhaustion to  
calculate the area under a  
parabola in his work  
Quadrature of the Parabola.  
Modern calculus was developed in 17th-century Europe by Isaac Newton and Gottfried Wilhelm  
Leibniz (independently of each other, first publishing around the same time) but elements of it  
first appeared in ancient Egypt and later Greece, then in China and the Middle East, and still later  
again in medieval Europe and India.  
Calculations of volume and area, one goal of integral calculus, can be found in the Egyptian  
Moscow papyrus (c. 1820 BC), but the formulae are simple instructions, with no indication as to  
how they were obtained.[7][8]  
Laying the foundations for integral calculus and foreshadowing the  
concept of the limit, ancient Greek mathematician Eudoxus of Cnidus  
(c. 390 – 337 BC) developed the method of exhaustion to prove the  
formulas for cone and pyramid volumes.  
During the Hellenistic period, this method was further developed by  
Archimedes (c. 287 – c. 212 BC), who combined it with a concept of  
the indivisibles—a precursor to infinitesimals—allowing him to solve  
several problems now treated by integral calculus. In The Method of  
Mechanical Theorems he describes, for example, calculating the  
center of gravity of a solid hemisphere, the center of gravity of a  
frustum of a circular paraboloid, and the area of a region bounded by a  
parabola and one of its secant lines.[9]  
The method of exhaustion was later discovered independently in  
China by Liu Hui in the 3rd century AD to find the area of a  
circle.[10][11] In the 5th century AD, Zu Gengzhi, son of Zu Chongzhi,  
established a method[12][13] that would later be called Cavalieri's  
principle to find the volume of a sphere.  
In the Middle East, Hasan Ibn al-Haytham, Latinized as Alhazen (c. 965 – c. 1040  AD) derived a  
formula for the sum of fourth powers. He used the results to carry out what would now be called an  
integration of this function, where the formulae for the sums of integral squares and fourth powers  
allowed him to calculate the volume of a paraboloid.[14]  
Ancient precursors  
Egypt  
Greece  
China  
Medieval  
Middle East  
India

Ibn al-Haytham,  
11th-century  
Arab  
mathematician  
and physicist  
Indian Mathematician and  
Astronomer Bhāskara II  
Bhāskara II (c.1114–1185) was acquainted with  
some ideas of differential calculus and suggested  
that the "differential coefficient" vanishes at an  
extremum value of the function.[15] In his  
astronomical work, he gave a procedure that  
looked like a precursor to infinitesimal methods.  
Namely,   
if   
   
then  
 This can be  
interpreted as the discovery that cosine is the  
derivative of sine.[16] In the 14th century, Indian  
mathematicians gave a non-rigorous method,  
resembling differentiation, applicable to some  
trigonometric   
functions.   
Madhava   
of  
Sangamagrama and the Kerala School of Astronomy and Mathematics stated components of  
calculus, but according to Victor J. Katz they were not able to "combine many differing ideas under  
the two unifying themes of the derivative and the integral, show the connection between the two,  
and turn calculus into the great problem-solving tool we have today".[14]  
Johannes Kepler's work Stereometria Doliorum (1615) formed the basis of integral calculus.[17]  
Kepler developed a method to calculate the area of an ellipse by adding up the lengths of many  
radii drawn from a focus of the ellipse.[18]  
Significant work was a treatise, the origin being Kepler's methods,[18] written by Bonaventura  
Cavalieri, who argued that volumes and areas should be computed as the sums of the volumes and  
areas of infinitesimally thin cross-sections. The ideas were similar to Archimedes' in The Method,  
but this treatise is believed to have been lost in the 13th century and was only rediscovered in the  
early 20th century, and so would have been unknown to Cavalieri. Cavalieri's work was not well  
respected since his methods could lead to erroneous results, and the infinitesimal quantities he  
introduced were disreputable at first.  
The formal study of calculus brought together Cavalieri's infinitesimals with the calculus of finite  
differences developed in Europe at around the same time. Pierre de Fermat, claiming that he  
borrowed from Diophantus, introduced the concept of adequality, which represented equality up to  
an infinitesimal error term.[19] The combination was achieved by John Wallis, Isaac Barrow, and  
James Gregory, the latter two proving predecessors to the second fundamental theorem of calculus  
around 1670.[20][21]  
The product rule and chain rule,[22] the notions of higher derivatives and Taylor series,[23] and of  
analytic functions[24] were used by Isaac Newton in an idiosyncratic notation which he applied to  
solve problems of mathematical physics. In his works, Newton rephrased his ideas to suit the  
mathematical idiom of the time, replacing calculations with infinitesimals by equivalent  
geometrical arguments which were considered beyond reproach. He used the methods of calculus  
to solve the problem of planetary motion, the shape of the surface of a rotating fluid, the oblateness  
of the earth, the motion of a weight sliding on a cycloid, and many other problems discussed in his  
Principia Mathematica (1687). In other work, he developed series expansions for functions,  
Modern

Gottfried Wilhelm Leibniz  
was the first to state clearly  
the rules of calculus.  
Isaac Newton  
developed the use of  
calculus in his laws of  
motion and universal  
gravitation.  
including fractional and irrational powers, and it was clear that he understood the principles of the  
Taylor series. He did not publish all these discoveries, and at this time infinitesimal methods were  
still considered disreputable.[25]  
These ideas were arranged into a true calculus of  
infinitesimals by Gottfried Wilhelm Leibniz, who  
was   
originally   
accused   
of   
plagiarism   
by  
Newton.[26]   
He   
is   
now   
regarded   
as   
an  
independent inventor of and contributor to  
calculus. His contribution was to provide a clear  
set of rules for working with infinitesimal  
quantities, allowing the computation of second  
and higher derivatives, and providing the  
product rule and chain rule, in their differential  
and integral forms. Unlike Newton, Leibniz put  
painstaking   
effort   
into   
his   
choices   
of  
notation.[27]  
Today, Leibniz and Newton are usually both  
given credit for independently inventing and  
developing calculus. Newton was the first to  
apply calculus to general physics. Leibniz developed much of the notation used in calculus  
today.[28]: 51–52  The basic insights that both Newton and Leibniz provided were the laws of  
differentiation and integration, emphasizing that differentiation and integration are inverse  
processes, second and higher derivatives, and the notion of an approximating polynomial series.  
When Newton and Leibniz first published their results, there was great controversy over which  
mathematician (and therefore which country) deserved credit. Newton derived his results first  
(later to be published in his Method of Fluxions), but Leibniz published his "Nova Methodus pro  
Maximis et Minimis" first. Newton claimed Leibniz stole ideas from his unpublished notes, which  
Newton had shared with a few members of the Royal Society. This controversy divided English-  
speaking mathematicians from continental European mathematicians for many years, to the  
detriment of English mathematics.[29] A careful examination of the papers of Leibniz and Newton  
shows that they arrived at their results independently, with Leibniz starting first with integration  
and Newton with differentiation. It is Leibniz, however, who gave the new discipline its name.  
Newton called his calculus "the science of fluxions", a term that endured in English schools into the  
19th century.[30]: 100  The first complete treatise on calculus to be written in English and use the  
Leibniz notation was not published until 1815.[31]  
Since the time of Leibniz and Newton, many mathematicians have contributed to the continuing  
development of calculus. One of the first and most complete works on both infinitesimal and  
integral calculus was written in 1748 by Maria Gaetana Agnesi.[32][33]  
In calculus, foundations refers to the rigorous development of the subject from axioms and  
definitions. In early calculus, the use of infinitesimal quantities was thought unrigorous and was  
fiercely criticized by several authors, most notably Michel Rolle and Bishop Berkeley. Berkeley  
famously described infinitesimals as the ghosts of departed quantities in his book The Analyst in  
Foundations

Maria Gaetana Agnesi  
1734. Working out a rigorous foundation for calculus occupied  
mathematicians for much of the century following Newton and  
Leibniz, and is still to some extent an active area of research today.[34]  
Several mathematicians, including Maclaurin, tried to prove the  
soundness of using infinitesimals, but it would not be until 150 years  
later when, due to the work of Cauchy and Weierstrass, a way was  
finally found to avoid mere "notions" of infinitely small quantities.[35]  
The foundations of differential and integral calculus had been laid. In  
Cauchy's Cours d'Analyse, we find a broad range of foundational  
approaches, including a definition of continuity in terms of  
infinitesimals, and a (somewhat imprecise) prototype of an (ε, δ)-  
definition of limit in the definition of differentiation.[36] In his work,  
Weierstrass formalized the concept of limit and eliminated infinitesimals (although his definition  
can validate nilsquare infinitesimals). Following the work of Weierstrass, it eventually became  
common to base calculus on limits instead of infinitesimal quantities, though the subject is still  
occasionally called "infinitesimal calculus". Bernhard Riemann used these ideas to give a precise  
definition of the integral.[37] It was also during this period that the ideas of calculus were  
generalized to the complex plane with the development of complex analysis.[38]  
In modern mathematics, the foundations of calculus are included in the field of real analysis, which  
contains full definitions and proofs of the theorems of calculus. The reach of calculus has also been  
greatly extended. Henri Lebesgue invented measure theory, based on earlier developments by  
Émile Borel, and used it to define integrals of all but the most pathological functions.[39] Laurent  
Schwartz introduced distributions, which can be used to take the derivative of any function  
whatsoever.[40]  
Limits are not the only rigorous approach to the foundation of calculus. Another way is to use  
Abraham Robinson's non-standard analysis. Robinson's approach, developed in the 1960s, uses  
technical machinery from mathematical logic to augment the real number system with  
infinitesimal and infinite numbers, as in the original Newton-Leibniz conception. The resulting  
numbers are called hyperreal numbers, and they can be used to give a Leibniz-like development of  
the usual rules of calculus.[41] There is also smooth infinitesimal analysis, which differs from non-  
standard analysis in that it mandates neglecting higher-power infinitesimals during  
derivations.[34] Based on the ideas of F. W. Lawvere and employing the methods of category  
theory, smooth infinitesimal analysis views all functions as being continuous and incapable of  
being expressed in terms of discrete entities. One aspect of this formulation is that the law of  
excluded middle does not hold.[34] The law of excluded middle is also rejected in constructive  
mathematics, a branch of mathematics that insists that proofs of the existence of a number,  
function, or other mathematical object should give a construction of the object. Reformulations of  
calculus in a constructive framework are generally part of the subject of constructive analysis.[34]  
While many of the ideas of calculus had been developed earlier in Greece, China, India, Iraq,  
Persia, and Japan, the use of calculus began in Europe, during the 17th century, when Newton and  
Leibniz built on the work of earlier mathematicians to introduce its basic principles.[11][25][42] The  
Hungarian polymath John von Neumann wrote of this work,  
Significance

The calculus was the first achievement of modern mathematics and it is difficult to  
overestimate its importance. I think it defines more unequivocally than anything else the  
inception of modern mathematics, and the system of mathematical analysis, which is its  
logical development, still constitutes the greatest technical advance in exact thinking.[43]  
Applications of differential calculus include computations involving velocity and acceleration, the  
slope of a curve, and optimization.[44]: 341–453  Applications of integral calculus include  
computations involving area, volume, arc length, center of mass, work, and pressure.[44]: 685–700   
More advanced applications include power series and Fourier series.  
Calculus is also used to gain a more precise understanding of the nature of space, time, and  
motion. For centuries, mathematicians and philosophers wrestled with paradoxes involving  
division by zero or sums of infinitely many numbers. These questions arise in the study of motion  
and area. The ancient Greek philosopher Zeno of Elea gave several famous examples of such  
paradoxes. Calculus provides tools, especially the limit and the infinite series, that resolve the  
paradoxes.[45]  
Calculus is usually developed by working with very small quantities. Historically, the first method  
of doing so was by infinitesimals. These are objects which can be treated like real numbers but  
which are, in some sense, "infinitely small". For example, an infinitesimal number could be greater  
than 0, but less than any number in the sequence 1, 1/2, 1/3, ... and thus less than any positive real  
number. From this point of view, calculus is a collection of techniques for manipulating  
infinitesimals. The symbols   
 and   
 were taken to be infinitesimal, and the derivative   
was their ratio.[34]  
The infinitesimal approach fell out of favor in the 19th century because it was difficult to make the  
notion of an infinitesimal precise. In the late 19th century, infinitesimals were replaced within  
academia by the epsilon, delta approach to limits. Limits describe the behavior of a function at a  
certain input in terms of its values at nearby inputs. They capture small-scale behavior using the  
intrinsic structure of the real number system (as a metric space with the least-upper-bound  
property). In this treatment, calculus is a collection of techniques for manipulating certain limits.  
Infinitesimals get replaced by sequences of smaller and smaller numbers, and the infinitely small  
behavior of a function is found by taking the limiting behavior for these sequences. Limits were  
thought to provide a more rigorous foundation for calculus, and for this reason, they became the  
standard approach during the 20th century. However, the infinitesimal concept was revived in the  
20th century with the introduction of non-standard analysis and smooth infinitesimal analysis,  
which provided solid foundations for the manipulation of infinitesimals.[34]  
Differential calculus is the study of the definition, properties, and applications of the derivative of a  
function. The process of finding the derivative is called differentiation. Given a function and a  
point in the domain, the derivative at that point is a way of encoding the small-scale behavior of  
Principles  
Limits and infinitesimals  
Differential calculus

Tangent line at (x0, f(x0)). The derivative f′(x) of  
a curve at a point is the slope (rise over run) of  
the line tangent to that curve at that point.  
the function near that point. By finding the  
derivative of a function at every point in its domain,  
it is possible to produce a new function, called the  
derivative function or just the derivative of the  
original function. In formal terms, the derivative is  
a linear operator which takes a function as its input  
and produces a second function as its output. This  
is more abstract than many of the processes studied  
in elementary algebra, where functions usually  
input a number and output another number. For  
example, if the doubling function is given the input  
three, then it outputs six, and if the squaring  
function is given the input three, then it outputs  
nine. The derivative, however, can take the squaring  
function as an input. This means that the derivative  
takes all the information of the squaring function—such as that two is sent to four, three is sent to  
nine, four is sent to sixteen, and so on—and uses this information to produce another function. The  
function produced by differentiating the squaring function turns out to be the doubling  
function.[28]: 32   
In more explicit terms the "doubling function" may be denoted by g(x) = 2x and the "squaring  
function" by f(x) = x2. The "derivative" now takes the function f(x), defined by the expression "x2",  
as an input, that is all the information—such as that two is sent to four, three is sent to nine, four is  
sent to sixteen, and so on—and uses this information to output another function, the function  
g(x) = 2x, as will turn out.  
In Lagrange's notation, the symbol for a derivative is an apostrophe-like mark called a prime.  
Thus, the derivative of a function called f is denoted by f′, pronounced "f prime" or "f dash". For  
instance, if f(x) = x2 is the squaring function, then f′(x) = 2x is its derivative (the doubling  
function g from above).  
If the input of the function represents time, then the derivative represents change concerning time.  
For example, if f is a function that takes time as input and gives the position of a ball at that time as  
output, then the derivative of f is how the position is changing in time, that is, it is the velocity of  
the ball.[28]: 18–20   
If a function is linear (that is if the graph of the function is a straight line), then the function can be  
written as y = mx + b, where x is the independent variable, y is the dependent variable, b is the y-  
intercept, and:  
This gives an exact value for the slope of a straight line.[46]: 6  If the graph of the function is not a  
straight line, however, then the change in y divided by the change in x varies. Derivatives give an  
exact meaning to the notion of change in output concerning change in input. To be concrete, let f  
be a function, and fix a point a in the domain of f. (a, f(a)) is a point on the graph of the function.  
If h is a number close to zero, then a + h is a number close to a. Therefore, (a + h, f(a + h)) is  
close to (a, f(a)). The slope between these two points is

The derivative f′(x) of a curve at a point is the  
slope of the line tangent to that curve at that point.  
This slope is determined by considering the  
limiting value of the slopes of the second lines.  
Here the function involved (drawn in red) is  
f(x) = x3 − x. The tangent line (in green) which  
passes through the point (−3/2, −15/8) has a  
slope of 23/4. The vertical and horizontal scales in  
this image are different.  
This expression is called a difference quotient. A line through two points on a curve is called a  
secant line, so m is the slope of the secant line between (a, f(a)) and (a + h, f(a + h)). The second  
line is only an approximation to the behavior of the function at the point a because it does not  
account for what happens between a and a + h. It is not possible to discover the behavior at a by  
setting h to zero because this would require dividing by zero, which is undefined. The derivative is  
defined by taking the limit as h tends to zero, meaning that it considers the behavior of f for all  
small values of h and extracts a consistent value for the case when h equals zero:  
Geometrically, the derivative is the slope of the tangent line to the graph of f at a. The tangent line  
is a limit of secant lines just as the derivative is a limit of difference quotients. For this reason, the  
derivative is sometimes called the slope of the function f.[46]: 61–63   
Here is a particular example, the derivative of the squaring function at the input 3. Let f(x) = x2 be  
the squaring function.  
The slope of the tangent line to the squaring  
function at the point (3, 9) is 6, that is to say, it is  
going up six times as fast as it is going to the right.  
The limit process just described can be performed  
for any point in the domain of the squaring  
function. This defines the derivative function of the  
squaring function or just the derivative of the  
squaring function for short. A computation similar  
to the one above shows that the derivative of the  
squaring function is the doubling function.[46]: 63   
A common notation, introduced by Leibniz, for the  
derivative in the example above is  
Leibniz notation

In an approach based on limits, the symbol ⁠dy  
dx⁠ is to be interpreted not as the quotient of two  
numbers but as a shorthand for the limit computed above.[46]: 74  Leibniz, however, did intend it to  
represent the quotient of two infinitesimally small numbers, dy being the infinitesimally small  
change in y caused by an infinitesimally small change dx applied to x. We can also think of ⁠d  
dx⁠ as a  
differentiation operator, which takes a function as an input and gives another function, the  
derivative, as the output. For example:  
In this usage, the dx in the denominator is read as "with respect to x".[46]: 79  Another example of  
correct notation could be:  
Even when calculus is developed using limits rather than infinitesimals, it is common to  
manipulate symbols like dx and dy as if they were real numbers; although it is possible to avoid  
such manipulations, they are sometimes notationally convenient in expressing operations such as  
the total derivative.  
Integral calculus is the study of the definitions, properties, and applications of two related  
concepts, the indefinite integral and the definite integral. The process of finding the value of an  
integral is called integration.[44]: 508  The indefinite integral, also known as the antiderivative, is  
the inverse operation to the derivative.[46]: 163–165  F is an indefinite integral of f when f is a  
derivative of F. (This use of lower- and upper-case letters for a function and its indefinite integral  
is common in calculus.) The definite integral inputs a function and outputs a number, which gives  
the algebraic sum of areas between the graph of the input and the x-axis. The technical definition  
of the definite integral involves the limit of a sum of areas of rectangles, called a Riemann  
sum.[47]: 282   
A motivating example is the distance traveled in a given time.[46]: 153  If the speed is constant, only  
multiplication is needed:  
But if the speed changes, a more powerful method of finding the distance is necessary. One such  
method is to approximate the distance traveled by breaking up the time into many short intervals  
of time, then multiplying the time elapsed in each interval by one of the speeds in that interval, and  
then taking the sum (a Riemann sum) of the approximate distance traveled in each interval. The  
Integral calculus

Integration can be thought of as measuring the  
area under a curve, defined by f(x), between  
two points (here a and b).  
A sequence of midpoint Riemann sums over a  
regular partition of an interval: the total area of  
the rectangles converges to the integral of the  
function.  
basic idea is that if only a short time elapses, then the  
speed will stay more or less the same. However, a  
Riemann sum only gives an approximation of the  
distance traveled. We must take the limit of all such  
Riemann sums to find the exact distance traveled.  
When velocity is constant, the total distance traveled  
over the given time interval can be computed by  
multiplying velocity and time. For example, traveling  
a steady 50  mph for 3 hours results in a total  
distance of 150 miles. Plotting the velocity as a  
function of time yields a rectangle with a height  
equal to the velocity and a width equal to the time  
elapsed. Therefore, the product of velocity and time  
also calculates the rectangular area under the  
(constant) velocity curve.[44]: 535  This connection  
between the area under a curve and the distance  
traveled can be extended to any irregularly shaped  
region exhibiting a fluctuating velocity over a given  
period. If f(x) represents speed as it varies over time,  
the distance traveled between the times represented  
by a and b is the area of the region between f(x) and  
the x-axis, between x = a and x = b.  
To approximate that area, an intuitive method would  
be to divide up the distance between a and b into  
several equal segments, the length of each segment  
represented by the symbol Δx. For each small  
segment, we can choose one value of the function  
f(x). Call that value h. Then the area of the rectangle with base Δx and height h gives the distance  
(time Δx multiplied by speed h) traveled in that segment. Associated with each segment is the  
average value of the function above it, f(x) = h. The sum of all such rectangles gives an  
approximation of the area between the axis and the curve, which is an approximation of the total  
distance traveled. A smaller value for Δx will give more rectangles and in most cases a better  
approximation, but for an exact answer, we need to take a limit as Δx approaches zero.[44]: 512–522   
The symbol of integration is   
, an elongated S chosen to suggest summation.[44]: 529  The definite  
integral is written as:  
and is read "the integral from a to b of f-of-x with respect to x." The Leibniz notation dx is intended  
to suggest dividing the area under the curve into an infinite number of rectangles so that their  
width Δx becomes the infinitesimally small dx.[28]: 44   
The indefinite integral, or antiderivative, is written:

Functions differing by only a constant have the same derivative, and it can be shown that the  
antiderivative of a given function is a family of functions differing only by a constant.[47]: 326  Since  
the derivative of the function y = x2 + C, where C is any constant, is y′ = 2x, the antiderivative of  
the latter is given by:  
The unspecified constant C present in the indefinite integral or antiderivative is known as the  
constant of integration.[48]: 135   
The fundamental theorem of calculus states that differentiation and integration are inverse  
operations.[47]: 290  More precisely, it relates the values of antiderivatives to definite integrals.  
Because it is usually easier to compute an antiderivative than to apply the definition of a definite  
integral, the fundamental theorem of calculus provides a practical way of computing definite  
integrals. It can also be interpreted as a precise statement of the fact that differentiation is the  
inverse of integration.  
The fundamental theorem of calculus states: If a function f is continuous on the interval [a, b] and  
if F is a function whose derivative is f on the interval (a, b), then  
Furthermore, for every x in the interval (a, b),  
This realization, made by both Newton and Leibniz, was key to the proliferation of analytic results  
after their work became known. (The extent to which Newton and Leibniz were influenced by  
immediate predecessors, and particularly what Leibniz may have learned from the work of Isaac  
Barrow, is difficult to determine because of the priority dispute between them.[49]) The  
fundamental theorem provides an algebraic method of computing many definite integrals—  
without performing limit processes—by finding formulae for antiderivatives. It is also a prototype  
solution of a differential equation. Differential equations relate an unknown function to its  
derivatives and are ubiquitous in the sciences.[50]: 351–352   
Calculus is used in every branch of the physical sciences,[51]: 1  actuarial science, computer science,  
statistics, engineering, economics, business, medicine, demography, and in other fields wherever a  
problem can be mathematically modeled and an optimal solution is desired.[52] It allows one to go  
from (non-constant) rates of change to the total change or vice versa, and many times in studying a  
Fundamental theorem  
Applications

The logarithmic spiral of the Nautilus  
shell is a classical image used to  
depict the growth and change  
related to calculus.  
problem we know one and are trying to find the other.[53]  
Calculus can be used in conjunction with other mathematical  
disciplines. For example, it can be used with linear algebra to  
find the "best fit" linear approximation for a set of points in a  
domain. Or, it can be used in probability theory to determine  
the expectation value of a continuous random variable given a  
probability density function.[54]: 37  In analytic geometry, the  
study of graphs of functions, calculus is used to find high points  
and low points (maxima and minima), slope, concavity and  
inflection points. Calculus is also used to find approximate  
solutions to equations; in practice, it is the standard way to  
solve differential equations and do root finding in most  
applications. Examples are methods such as Newton's method,  
fixed point iteration, and linear approximation. For instance,  
spacecraft use a variation of the Euler method to approximate curved courses within zero gravity  
environments.  
Physics makes particular use of calculus; all concepts in classical mechanics and electromagnetism  
are related through calculus. The mass of an object of known density, the moment of inertia of  
objects, and the potential energies due to gravitational and electromagnetic forces can all be found  
by the use of calculus. An example of the use of calculus in mechanics is Newton's second law of  
motion, which states that the derivative of an object's momentum concerning time equals the net  
force upon it. Alternatively, Newton's second law can be expressed by saying that the net force  
equals the object's mass times its acceleration, which is the time derivative of velocity and thus the  
second time derivative of spatial position. Starting from knowing how an object is accelerating, we  
use calculus to derive its path.[55]  
Maxwell's theory of electromagnetism and Einstein's theory of general relativity are also expressed  
in the language of differential calculus.[56][57]: 52–55  Chemistry also uses calculus in determining  
reaction rates[58]: 599  and in studying radioactive decay.[58]: 814  In biology, population dynamics  
starts with reproduction and death rates to model population changes.[59][60]: 631   
Green's theorem, which gives the relationship between a line integral around a simple closed curve  
C and a double integral over the plane region D bounded by C, is applied in an instrument known  
as a planimeter, which is used to calculate the area of a flat surface on a drawing.[61] For example,  
it can be used to calculate the amount of area taken up by an irregularly shaped flower bed or  
swimming pool when designing the layout of a piece of property.  
In the realm of medicine, calculus can be used to find the optimal branching angle of a blood vessel  
to maximize flow.[62] Calculus can be applied to understand how quickly a drug is eliminated from  
a body or how quickly a cancerous tumor grows.[63]  
In economics, calculus allows for the determination of maximal profit by providing a way to easily  
calculate both marginal cost and marginal revenue.[64]: 387   
Glossary of calculus  
List of calculus topics  
See also

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consequently they left the foundations of analytical geometry and the infinitesimal calculus  
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nineteenth century, showed how to establish calculus without infinitesimals, and thus, at last,  
made it logically secure. Next came Georg Cantor, who developed the theory of continuity and  
infinite number. "Continuity" had been, until he defined it, a vague word, convenient for  
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